INTERPRETATION OF OPTICAL MEASUREMENTS IN CHANNEL AND SHOCK-WAVE EXPANSION SPEEDS FOR A HIGH-VOLTAGE DISCHARGE IN A LIQUID

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1. **Introduction.** Electrical discharges in liquids have been examined repeatedly, but up to now there has been no satisfactory model that can be used with given discharge-circuit parameters (C, L, U_0, R_0, l) to calculate the hydrodynamic flow. The basic reason is the inadequate development of the theory of dense nonideal low-temperature plasma in channels with inhomogeneity over the cross section and variable numbers of particles. Progress in this area is held back by the restricted scope for experimental investigation, particularly as regards the voltage and current in the discharge, the channel geometry, and sometimes the surface temperature.

Important information is provided by pressure determination in the channel. At present, the only method is by the interpretation of optical measurements on the wall path. The recording system of Fig. 1 enables one to record the size of the channel in transmitted light together with shock-wave propagation. A characteristic SFR recording (Fig. 2) shows the primary wave propagating at acoustic speed. The main shock wave is produced at a certain distance from the channel, and up to this time there is simply a nonstationary high-pressure compression wave.

The pressures arising in discharge channels have frequently been measured [1-3] via the assumption of a constant expansion velocity in the first quarter period. Then there is a self-modeling solution to the one-dimensional gas-dynamic problem [4-6]. For expansion speeds $\dot{r}_c \ll c_0$, the approximation of linear acoustics has been used [7, 8], which incorporates the velocity variation. Recently, attempts have been made [9-11] to calculate the liquid flows around sparks by numerical methods.

2. **Formulation.** A numerical method was used to construct the flow picture from the kinematics of the expansion. Usually, the density of a plasma is less by two orders of magnitude than the density of water, and therefore the possible error in determining the channel radius arising from the flow of material through the boundary is $\Delta r/r < 0.01$. Therefore, the piston permeability may be neglected from the viewpoint of the hydrodynamic flow.

We calculated the one-dimensional flow of a liquid initially at rest on the expansion of an impermeable cylindrical piston. The Neumann–Richtmyer [12] artificial-viscosity method was used to calculate the flow with shock waves by means of a cross-type difference scheme. The equation of state for water is $p = A(\rho/\rho_0)^m - B$ up to 20 kbar and was approximated by the expression

$$p = 0.001 + z(21.77 + z(66.95 + z(114.9 + z(119.2 + z(75 + 27\,z))))),$$

where $z = \rho/\rho_0 - 1$, which reduces the run time considerably. See [13] for details of the calculation method and approximation of the equation of state. The Walker–Sternberg [14] equation of state was used for high pressure.

Reliable determination of the time dependence of the pressure in the channel requires correction for the distortion of the recorded size arising from the change in the refractive index of water with pressure, and also the error in measuring the expansion of the channel due to the small size at the initial stage ($t \leqslant 1$ µsec). We consider the effects of each of these factors.

3. **Optical Distortions Due to Refractive-Index Change in the Water.** The recording system (Fig. 1) shows that the following formula applies [15] for the visible shadow radius of the channel:

$$r_* = r_c\left(\frac{n}{n_0} + \frac{\Delta n}{n_0} + \frac{n}{n_0}\frac{x_1}{r_c} + \frac{\Delta n}{n_0}\frac{x_1}{r_c}\right), \tag{3.1}$$

where n_0 is the refractive index of water under normal conditions, $n = n(p)$ is the refractive index at a given pressure and normal temperature, Δn is the change in refractive index at a given pressure due to heating (the values of n and Δn are taken for $r = y_0$), and $x_1 = y_0 - r_c$ is the minimum distance from the boundary of the channel to the path of the ray that indicates the boundary in shadow recording. All rays passing near the boundary of the channel are highly deviated on account of the gradients in the refractive index due to heating, and they fall outside the aperture of the recording instruments ($\alpha > \alpha_*$). Data on the absorption spectrum of water [16-18] were used in deriving upper bounds to the heating of the

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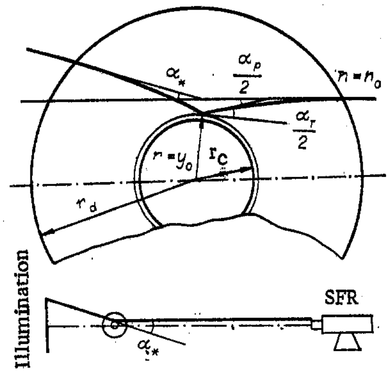

Fig. 1

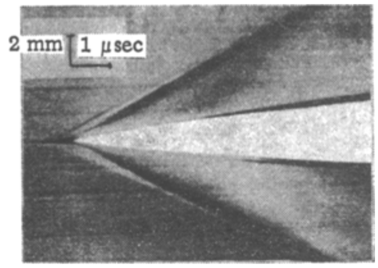

Fig. 2

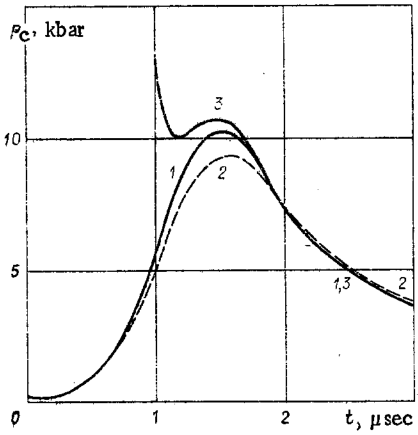

Fig. 3

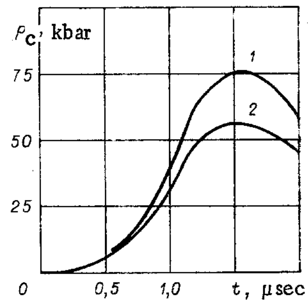

Fig. 4

liquid arising from the absorption of ultraviolet radiation and from conduction. The thickness of the heated layer was $\Delta x \ll r_c$ in discharges with channel temperatures up to 50,000° within the time range of interest. Then the beam deviation due to the refractive-index gradients arising from heating is

$$\frac{|\Delta n|}{n} = 1 - \cos\left(\frac{\alpha_T}{2}\right) \approx \frac{\alpha_*^2}{8},$$

since $\alpha_* \approx \alpha_T$.

In choosing the maximum possible angle of illumination α_* in the range $10° \leqslant \alpha_* \leqslant 20°$ for the ray detecting the boundary we obey the condition $|\Delta n|/n \leqslant 0.02$; in that case the ray passes in a comparatively cool region $\Delta T \ll 1000°$, and therefore the density dependence of the refractive index can be derived from the following expression [19]:

$$n = 1 + 0.334\rho. \tag{3.2}$$

The $\rho(p, T)$ dependence from [20-24] can be used to show that $|\Delta n|/n \approx 0.02$ requires heating by about 100°. This corresponds to $x_1 < 0.01\, r_c$ from estimates of the heating, and with the optimal choice of α_* it follows from (3.1) that with an error less than 2%

$$r_* = r_c n(p)/n_0. \tag{3.3}$$

Therefore, the measured radius is greater than the true radius arising from the optical distortion caused by the elevated pressure near the channel.

If the boundary condition for the piston is put as $r_0(t) = kr_*(t)$, where $k = n_0/(1 + 0.334\rho)$, and ρ is the density of the water near the boundary of the channel outside the heating zone, then the optical distortions will balance out in calculating the flow. The boundary condition was approximated via the inexplicit scheme $r_0^{n+1} = k_{1/2}^{n+1} r_*^{n+1}$. The system of inexplicit equations for the first cell in each time layer was solved by iteration. The channel expansion indicated by the set of experimental points $r_i(t_i)$ was approximated by cubic spline interpolation. This provided continuity in the first and second derivatives at the interpolation nodes, which is necessary in calculating the hydrodynamic flows when there is gradual energy deposition in the discharge channel.

TABLE 1

δ, %	20	12	4	2	1
k	1,7	2	3	4	6

TABLE 2

t, μsec	E, J/cm	ε, %	δ, %	k	δ, % from (4.1)
1,0	0,58	100	128	1	—
1,2	2,19	26	18	1,5	>20
1,4	5,78	10	6	2,2	10—11
1,6	11,5	5	2,9	2,8	5—6
1,8	18,5	3	1	3,5	3

Figure 3 shows the channel pressure as a function of time without correction for the optical distortion (curve 1) and with such correction (curve 2) for one of the observed expansion laws. It is clear that the pressure amplitude in the first case is exaggerated by 10%. It follows from (3.3) that the velocity and the modulus of the acceleration are also overestimated. However, as the acceleration is negative, the corrections to the pressure appear with opposite signs, i.e., they may balance out at some instant (Fig. 3).

Figure 4 shows calculations for high pressures as derived from the model expansion law (curves 1 and 2 denote the same as in Fig. 3). At pressures of the order of 50 kbar, which can be attained with current capacitor batteries [25], the optical distortions result in considerable discrepancies between the results.

It is important to correct for the distortions in determining the plasma conductivity from the current and voltage. One then uses data on the section of the channel derived from the optical measurements. Consequently, the relative error in determining σ will be $\Delta\sigma/\sigma = (1 - k)^2$, i.e., 12% even at a pressure of about 10 kbar.

4. Effects of Inaccurate Determination of Expansion at the Initial Stage on the Pressure Calculation. While the channel is small, there is a marked error in measuring the radius. However, only a small fraction of the total energy is transmitted to the liquid in this time, and therefore there will only be a slight effect on the parameters from error in determining the expansion in the initial stage when there is an ongoing energy transfer for some time t_*. Reliable determination of the pressure from the expansion kinematics requires an exact knowledge of the maximum possible error introduced up to time t_* by the uncertainty in the initial stage.

A first estimate has been made [13] from the self-modeling solution for a cylindrical piston beginning to expand from $r = r_0$ with a constant speed u_0. Then we get an accuracy $\delta = (p_c - p_{ca})/p_{ca}$ for the pressure in the channel at time t_* as defined from $r_c(t_*)/r_0 = k(\delta)$, where $k(\delta)$ is a tabulated function (Table 1). The latter was derived from a series of numerical experiments with u_0 varying in the range 0.05-1 km/sec.

We now examine the quantitative dependence of the solution on the expansion law in the initial stage with a variable speed for the example of Fig. 3 (curve 1). Up to $t = t_0 = 1$ μsec, the motion of the boundary of the channel is not known exactly. A crude approximation for the initial stage is $r_c(t) = r_c(t_0)$ for $t < t_0$, and then we get the result shown by curve 3 for the pressure p'_c in a channel. The relative error is $\delta(t) = (p'_c - p_c)/p_c$ at 0.8 μsec from the start of the motion (i.e., at $t = 1.8$ μsec) and is then less than 1%. We can compare the results (Table 2) with the estimate of $\delta(t_*)$

$$r_c(t_*)/r_c(t_0) = k(\delta). \tag{4.1}$$

The energy meaning of δ of (4.1) will become clear on comparison with $\epsilon = E_0/E \cdot 100\%$, where E_0 and E are the energies transmitted to the liquid up to times t_0 and $t > t_0$.

Therefore, if there is ongoing energy transfer, there can be a relative error in determining the channel pressure from the expansion kinematics at time t_* due to the uncertainty over the initial stage that is less than the estimate of $\delta(t_*)$ given by (4.1).

5. Determination of Shock-Wave Pressure from Speed. For the case of curve 1 in Fig. 3 we calculated the pressure fields for various instants (Fig. 5); curves 1-5 relate to times of 0.4; 0.8; 1.2; 1.6; 2.0 μsec. As we assume that $\dot{r}_c(0) \neq 0$, a weak shock wave emerges from the channel at the initial instant, whose front propagates almost with the speed of sound. At $t \approx 2$ μsec, the region of smooth flow at a certain distance from the channel gives rise to the main shock wave, while before this there is simply a nonstationary high-pressure compression wave. The results were used in constructing an $r - t$ diagram for the process, i.e., a simulation of the shadow recording.

The entire region of elevated pressure was split up into N concentric rings in order to calculate the deviation angles [26], for each of which it was assumed that the refractive index varied only slightly and the refractive index gradient was constant. Then

$$\alpha_i = 2y_i \sum_{j=i+1}^{N} \left(-\frac{1}{n_j}\frac{\partial n_j}{\partial r}\right) \int_{r_{j-1}}^{r_j} \frac{dr}{\sqrt{r^2 - y_i^2}} + \theta_i, \quad \theta_i = 2\frac{\Delta n}{n_0}\frac{z_i}{\sqrt{1 - z_i^2}},$$

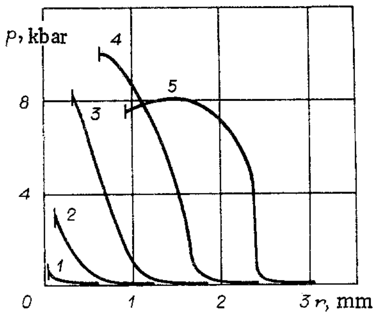

Fig. 5

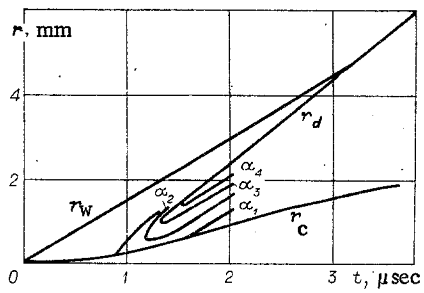

Fig. 6

where $z_i = y_i/r_d$; Δn is the discontinuity in the refractive index at the shock-wave front, and y_i is the minimum distance from the center of symmetry to the paths of ray i. We use (3.2) for the ρ dependence of n.

The r – t diagram (Fig. 6) shows the following: expansion path r_c of the channel, the first weak shock wave r_w, the main shock wave r_d, and the line for equal ray deviation in the compression wave ($\alpha_1 = 0.06$; $\alpha_2 = 0.1$; $\alpha_3 = 0.14$; $\alpha_4 = 0.2$ rad), which denote given degrees of film blackening in relation to the angular sensitivity of the shadow recording. It is impossible to use high sensitivity because the compression wave may then be taken as a shock wave. For example, a photoscan has been given [27] of the explosion of a wire in water with high sensitivity in schlieren photography, which went with an erroneous interpretation on which the pressure at the front of the apparent shock wave was calculated via the Rankin-Hugoniot relations.

Figure 6 shows that we should take $\alpha_* \geqslant 0.2$ rad at least, which falls in the range of optimal values of α_* in channel radius determination.

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